Delay induces motion of multipeak localized structures in cavity semiconductors

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ABSTRACT

We consider a broad area vertical-cavity surface-emitting laser(VCSEL) subject to injection and to time-delayed feedback. We show that near the nascent optical bistability regime, the space-time dynamics of this device is described by a generalized Swift-Hohenberg equation with delay. We classify different regime of stability of the homogeneous steady states in terms of dynamical parameters. We show that the delay modifies strongly the stability domain of both periodic and localized structures solutions. Finally, we show that the delay feedback induces a spontaneous motion of bright peaks in one and in two-dimensional transverse plane. Bifurcation diagram associated with these localized structures is constructed.

Keywords: VCSEL, nonlinear structures, delay differential equations on spatially extended systems

1. INTRODUCTION

Localized structures consist of bright of dark pulses which can be either independent or randomly distributed in space. They occur in various fields of nonlinear science, such as chemistry , plant ecology, and optics.[1,2] In all systems, despite the different physical origin, localized structures appear when the modulational or Turing instability is subcritical. When they are well separated from each other, localized peaks are independent or randomly distributed in the transverse plane. However, when they are close to each other they start to interact via their oscillating, exponentially decaying tails. This interaction leads to clustering phenomenon.[4] Localized structures attract growing interest in optics due to potential applications for all-optical control of light, optical storage, and information processing.[3,5]

Localized structures can move along one or more spatial dimensions with a constant velocity. Different mechanisms have been identified to be responsible for the motion of localized structures. These mechanisms are vorticity,[6] finite relaxation rates,[7–9] phase gradient,[10] so-called Ising-Bloch transition,[11–13] walk-off or convection or symmetry breaking due to off-axis feedback,[14] resonator detuning,[15] or a Hopf-Turing interacting bifurcations.[16] More recently, the inclusion of the delayed feedback in the dynamics of spatially extended systems that constitutes a robust and controllable mechanism allows for the mobility of localized structures. This motion under the effect of frequency selective delayed feedback has been observed experimentally[17] and theoretically.[18–20] When the delayed feedback is not frequency selective, i.e., regular, it has been shown that this effect leads to a spontaneous motion of localized structures.[21,22]

In this paper, we investigate the effect of a regular delayed feedback on the mobility of 2D localized structures in broad area Vertical-Cavity Surface Emitting Lasers (VCSEL). The delayed feedback is modeled by an external mirror located at a distance L from the VCSEL output facet. This completes a previous paper where the analysis was performed in strictly one transverse dimension.[22] We show that the dynamical behavior of modulational instability and localized structures formation is strongly affected by the delay. We provide a classification of

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	$a-25/48<0$	$a-25/48>0$
$p>\eta, d>0$	$]-\infty, f_{M-}]$	$\mathbb{R}-[f_{M+}, f_{M-}]$
$p>\eta, d<0$	$\mathbb{R}$	$\mathbb{R}-[f_{M+}, f_{M-}]$
$p<\eta-12d^2/25, d<0$	$]-\infty, f_{L-}]$	$\mathbb{R}-[f_{L+}, f_{M-}]$
$\eta-12d^2/25<p<\eta, d<0$	$]-\infty, f_{M+}]$	$\mathbb{R}-[f_{M+}, f_{M-}]$
$p<\eta-12d^2/25, d>0$	$]-\infty, f_{L+}]\cup[f_{L-}, f_{M+}]$	$\mathbb{R}-[f_{L+}, f_{M-}]$
$\eta-12d^2/25<p<\eta, d>0$	$]-\infty, f_{L+}]\cup[f_{M-}, f_{M+}]$	$\mathbb{R}-[f_{M+}, f_{M-}]$

Table 1. Classification of various stability domains for the steady homogeneous state $\bar{f}$ as a function dynamical parameters of the modified delayed Swift-Hohenberg equation (1)

different regimes of stability of the homogeneous steady states in terms of dynamical parameters. Finally, we describe the two-dimensional moving localized structures for this system. The paper is organized as follows: in the section 2 we introduce the generalized delayed Swift-Hohenberg equation describing the vicinity of the critical point associated with bistability, and we present the linear stability of the trivial states. The results are presented in section 3. We conclude in section 4.

2. NASCENT BISTABILITY REGIME AND LINEAR STABILITY ANALYSIS

We consider a VCSEL with optical injection and delayed feedback.[22] Our aim in this section is to present an order parameter equation which describes this device in limiting situations (i) near the critical point associated with bistability and (ii) near the long wavelength pattern forming regime. In this regime, the dynamics of VCSEL's exhibit a slowing down and the space-time dynamics can be described by the modified Swift-Hohenberg equation with a delay term[23]

$$\frac{\partial f}{\partial t} = y - f\left(p + f^2\right) + \left(d - \frac{5f}{2}\right)\nabla_\perp^2 f - a\nabla_\perp^4 f - 2(\nabla_\perp f)^2 + \eta f(t-\tau), \tag{1}$$

where f is the deviation of the electric field from its value at the onset of the bistability. y, p and d are the deviations from the injected field, the pump parameter and the diffusion coefficient of the carriers at the critical point, respectively. The parameter a is $a = (1-\alpha^2)/(4\alpha^2)$ where α is the linewidth enhancement factor. The Laplace operator acting on the transverse plane $\mathbf{r} = (\mathbf{x}, \mathbf{y})$ is $\nabla_\perp^2 = \partial_{xx}^2 + \partial_{yy}^2$. The feedback is characterized by the time-delay τ, and feedback strength η, while its phase is fixed to zero for simplicity. Without the delayed feedback, we recover the modified Swift-Hohenberg equation derived in.[24]

The homogeneous steady state solutions $\bar{f}$ of Equation (1) are solutions of $y = \bar{f}\left(p - \eta + \bar{f}^2\right)$. The system is monostable when $p > \eta$. If $p < \eta$, there is bistability with limit points $y_{L\pm} = f_{L\pm}\left(p - \eta + f_{L\pm}^2\right)$ with $f_{L\pm} = \pm\sqrt{(\eta - p)/3}$. We consider a linear deviation around the homogeneous state $\bar{f}$ proportional to $e^{\lambda t + \mathbf{k}\cdot\mathbf{r}}$, where $\mathbf{r} = (x, y)$ and $\mathbf{k_M}$ is the transverse wavevector. The transcendental characteristic equation reads

$$\lambda = -\left(3\bar{f}^2 + p\right) + \left(\frac{5\bar{f}}{2} - d\right)k^2 - ak^4 + \eta e^{-\lambda\tau}. \tag{2}$$

Turing instabilities correspond to the occurrence of zero real root $\lambda = 0$ and $\partial\lambda/\partial k = 0$. From equation (2), we see that the spectrum possesses a maximum nonzero wavenumber k only if $\bar{f} > 2d/5$. In that case we get $k_M^2 = (5\bar{f} - 2d)/(4a)$, and

$$\lambda_M = \lambda(k_M) = \frac{1}{4a}\left[12\left(\frac{25}{48} - a\right)\bar{f}^2 - 5d\bar{f} + d^2 - 4pa\right] + \eta e^{-\tau\lambda_M}. \tag{3}$$

The spontaneous pattern forming process requires $f_{M\pm} > 2d/5$ to have $k_M^2 > 0$. A modulational instability exists if $\lambda_M = 0$. This occurs at $\bar{f} = f_{M\pm}$ with

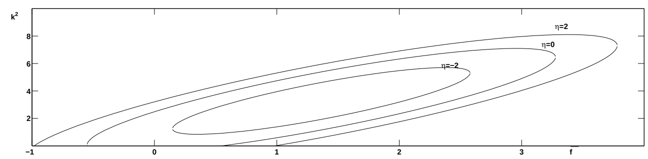

Figure 1. Marginal stability curves in the plane $(\bar{f}, k^2)$. Parameters are $p = -0.9$, $d = -1.5$ and $a = 0.75$. The strength of the feedback vary.

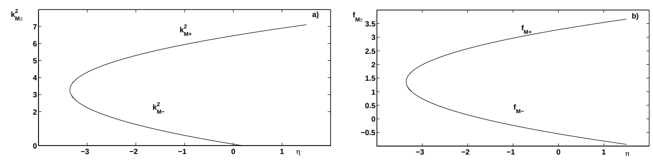

Figure 2. Critical wavenumbers (a) and thresholds (b) associated with modulational instabilities as a function of the strength of the delay. These curves correspond to the plot of Eq. (5) and Eq. (4), respectively. Parameters are $p = -0.9$, $d = -1.5$ and $a = 0.75$

$$f_{M\pm} = \frac{2}{25 - 48a}\left[5d \pm 2\sqrt{a(12d^2 + (p - \eta)(25 - 48a))}\right] \tag{4}$$

$$k^2_{M\pm} = \frac{5f_{M\pm} - 2d}{4a}. \tag{5}$$

From equation (3), we see that for $\bar{f}$ large and $a < 25/48$, all homogeneous steady states are unstable. On the basis of the sign of $a - 25/48$, the system could have zero, one or two modulational instability points. Classification of different scenarios leading to the instability of the homogeneous steady state are summarized in Table 1. An example of marginal instability curves for different value of the feedback strength is plotted in Fig. (1). The critical points as well as the critical wavenumbers associated with two modulational instabilities are plotted in the Fig. (2).

A traveling wave instability occurs when a pair of complex conjugate eigenvalues have a zero real part, i.e., $\lambda = \pm i\Omega$. This instability occurs when

$$\eta\cos(\Omega\tau) = (3\bar{f}^2_{TW\pm} + p) + (d - \frac{5\bar{f}_{TW\pm}}{2})k^2_{TW\pm} + ak^4_{TW\pm} \tag{6}$$

$$\eta\sin(\Omega\tau) = -\Omega \tag{7}$$

Classification of the traveling wave instabilities leading to the motion of periodic structures is reported in.[23] In what follows, we will focus on the spatial localization of light leading to the formation of both stationary and moving localized structures.

3. MOTION OF LOCALIZED STRUCTURES

Stationary localized structures have been observed in broad area Vertical-Cavity Surface Emitting Lasers with optical injection.[5] They are homoclinic solutions of Eq. (1) with $\partial f/\partial t = 0$, and they occur in the pinning region of parameters where the homogeneous steady states and the periodic solutions are both linearly stable. In the absence of delay feedback, localized structures are stationary objects. However when the delay feedback strength exceeds a threshold given by $\eta\tau = -1$,[21] a single localized structure starts to move in an arbitrary direction as shown in the left side space-map of Fig. (3). This is due to the isotropy in space. Another example of two bounded localized structures that are separated initially by some distance: they start to repeal each other, and exhibit a motion in opposite directions. This behavior is illustrated in the right side space-map of Fig. (3). The numerical simulations have been carried out using periodic boundary conditions. To characterize better these solutions, we construct a bifurcation diagram where we plot together with the homogeneous steady

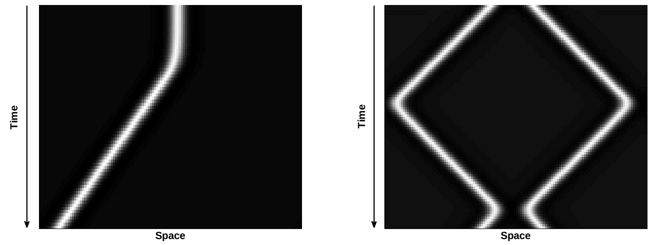

Figure 3. Space time map of a single (left) and of a double (right) peaked localized structure solutions of Eq. (1). Parameters are $p = -0.9$, $d = -1.5$, $y = -0.5$, $\eta = -0.15$, and $\tau = 15$.

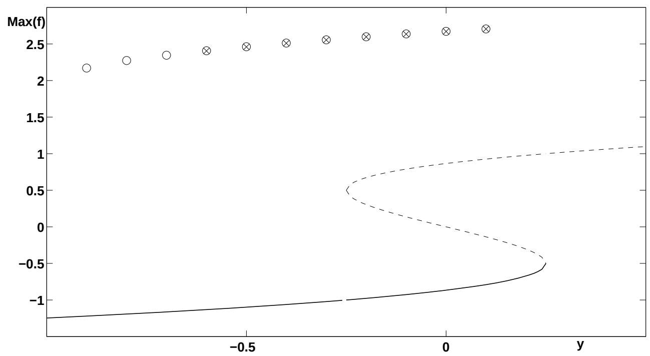

Figure 4. Bifurcation diagram associated with stationary and moving localized structures. Circles indicate maxima of stationary LS's obtained for $\tau = 0$, and crosses indicate maxima of moving LS's obtained for $\tau = 15$. The full and broken lines indicate stable (unstable) homogeneous steady states as a function of input optical field. Parameters are $\eta = -0.15$, $p = -0.9$, $d = -1.5$, $a = 0.75$. At large amplitude, the upper homogeneous steady state becomes stable at $\bar{f} = 3.5$.

states, the maximum amplitude associated with both stationary and moving localized structures as a function of the input field amplitude (see Fig. 4). The circles indicate the maximum amplitude of the stationary localized structures and the crosses represent the maximum of the moving ones. From this figure we see that moving localized structures have a smaller stability domain than the stationary ones, while their amplitudes remain the same for fixed values of parameters.

In the 2D setting, the number of localized peaks is obviously much larger than in 1D. In the subcritical domain involving hexagonal structures and a uniform solution which are both stable, a large variety of localized structures can be generated. The number and the position of localized peaks is solely determined by the initial condition. A single peak localized structure submitted to delayed feedback moves in an arbitrary direction [see Fig. (5a,b)]. If we modify the initial condition, we can generate in principle any number of LS's. An example of four peaks solution is shown in Fig. (5c,d) where peaks repeal each other and motion occurs in opposite directions. However, for a cluster involving large number of peaks, the motion of peaks occur in the same direction. This behavior is illustrated in Fig. (6).

4. CONCLUSION

We have described the space-time dynamics of a broad area vertical-cavity surface-emitting laser(VCSEL) subject to injection and to time-delayed feedback near the critical point associated with bistability and close to large wavelength pattern forming process. In this double limit, the dynamics of the VCSEL is described by the modified generalized Swift-Hohenberg with delay. We show that the delay feedback strongly affects the modulational instability by modifying the threshold as well as the wavelength. We show also that the modified Swift-Hohenberg equation supports stationary and moving localized structures in 1D and 2D. We construct the bifurcation diagram associated with moving localized structures.

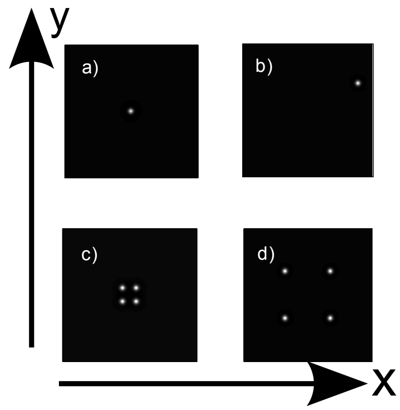

Figure 5. a) and b) represent the evolution of a single peak localized structure in the generalized Swift Hohenberg equation with a delay term. c) and d) represent the evolution of a multipeak solution: peaks repel each other. Parameters are $y = -0.5, p = -0.9, d = -1.5, a = 0.75, CELX = 128, CELY = 128, dt = 0.001$ and $dr = 0.4$ the distance between two adjacent cells. $CELX$ and $CELY$ are respectively the number of cells in the X and Y direction.

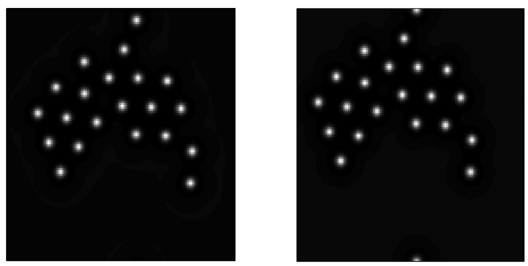

Figure 6. Motion of cluster with large number of peaks subjected to delayed feedback. The two figures show the transverse profile f solutions of equation Eq. (1) obtained at different time evolution. Same parameters as in Fig. (5).

5. ACKNOWLEDGEMENTS

M.T. is a Research Associate with the Fonds de la Recherche Scientifique F.R.S.-FNRS, Belgium. A.V. is supported by SFB 787 project of the DFG, EU FP7 Marie Curie Initial Training Network, Grant No. 264687, and the Programme "Research and Pedagogical Cadre for Innovative Russia", grant 2011-1.5-503-002-038.

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