Self-organized light bullets in type-I intracavity second harmonic generation

S. Blaha[1]*, E. Averlant[1,2], K. Panajotov[2,3], and M. Tlidi[1]

[1]Optique Nonlinéaire Théorique(ONT), Université Libre de Bruxelles (U.L.B.), CP 231, Campus Plaine, 1050 Brussels, Belgium

[2]Department of Applied Physics and Photonics (IR-TONA), Vrije Universiteit Brussels, Pleinlaan 2, B-1050 Brussels, Belgium

[3]Institute of Solid State Physics, Tzarigradsko Chaussee blvd. 1784, Sofia, Bulgaria

ABSTRACT

We study the formation of three-dimensional structures in type-I intracavity second harmonic generation model where polarization degrees of freedom due to the birefringence of the $\chi^{(2)}$ crystal is not considered. The device consists of an optical cavity filled with a quadratic nonlinear material and driven by an external beam at the fundamental frequency. The transmitted part of this field is coupled into the cavity where it undergoes second-harmonic conversion. These dissipative structures consist of regular or localized 3D lattices of bright spots travelling at the group velocity of light in the material. We show evidence of stable three dimensional structures such as stripes and cylinders.

Keywords: second harmonic generation, localized structures, pattern formation in spatially extended systems

1. INTRODUCTION

Dissipative structures in far from equilibium systems can be either periodic or localized in space. They have been observed experimentally in various nonlinear chemical, biological, hydrodynamical, and optical systems (see recent overviews on this issue[1–4]). They arise through nonlinear light-matter interaction in the presence of two-dimensional diffraction[5–12] or chromatic dispersion[13–15] and nonlinearity.

When the 2D diffraction and chromatic dispersion have the same influence in the nonlinear resonator, three-dimensional structures are formed. In free propagation in a Kerr medium, the electric field enveloppe is described by a nonlinear Schrodinger equation. However, it is well known that this equation in more than one dimension suffers from the collapse phenomenon. Several mechanisms can lead to stabilization of three dimensional confinement of light during propagation in nonlinear media. In particular, to avoid the problem of three-dimensional beam collapse, several studies incorporate a saturable nonlinearity[16] or an optical cavity,[17–20] or take place in the framework of the optical parametric oscillator pure from any modulational instability.[21] Numerical simulations reveal the existence of stationary bell-shaped 3D structures for both anomalous and normal chromatic dispersion regimes.[22] More recently, the combined influence of inhomogeneous diffraction/dispersion in the presence of the harmonic and parity-time symmetric potentials has been predicted to stabilize 3D structures.[23]

The spontaneous formation of three-dimensional structures in driven nonlinear optical cavity has been the subject of intense activity. They arise through nonlinear light-matter interaction in the presence of two-dimensional diffraction and dispersion, and nonlinearity. In addition, dissipation plays an important role in the stabilisation of such structures. An analytic study of the 3D Lugiato-Lefever model describing a coherently driven passive ring cavity filled with a Kerr medium has predicted, in the anomalous dispersion regime, the predominance of body-centered cubic (bcc) patterns in the cavity field over a large variety of other 3D solutions with less symmetry. The bcc pattern corresponds to a solution characterized by six different wave vectors. Soon after, analytical and numerical studies have shown evidence of bcc dissipative crystal in type-II second-harmonic generation. In addition we have shown that 3D hexagonal and lamella patterns have overlapping finite domains of stability, while the rhombic and face-centered cubic (fcc) lattices are unstable. This process involves the field polarization

*to whom correspondence should be addressed: sblaha@ulb.ac.be

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degrees of freedom due to the birefringence of the quadratic crystal. For this system, the analysis was restricted to the nascent optical bistability regime described by the Swift-Hohenberg equation.[24]

We investigate the formation of three dimensional structures in a nonlinear optical system that constitutes a paradigm for passive driven-damped coupled oscillators, namely, intracavity second-harmonic generation (nonlinear interaction between two resonant optical fields). This process takes place in an optical cavity filled with a quadratically nonlinear material. An external beam at the fundamental frequency is coupled into the cavity where it undergoes second-harmonic conversion. We consider a combined influence of 2D diffractive and dispersive type one second harmonic generation. We focus on regimes where one or more steady states exhibit a three dimensional pattern forming instability. The origin of this instability is the competition between the combined action of both 2D diffraction and chromatic dispersion with a quadratic nonlinearity. In this paper, we have studied the full type-I second harmonic generation assuming the mean field approximation. The 3D dissipative structures found are robust since they are generated spontaneously from a weak initial noise.

2. MODEL EQUATIONS

We consider an optical ring cavity filled with a quadratic nonlinear material where the second harmonic generation process takes place, i.e., two photons with frequency ω are absorbed by the nonlinear medium and only one photon with frequency 2ω is emitted. We limit our analysis to the phase matching of type I where the SHG process does not involve polarization degrees of freedom due to the birefringence of the $\chi^{(2)}$ crystal. We assume the validity of the mean field approximation (high finesse cavity) which oscillates on only one longitudinal mode. Taking into account diffraction and chromatic dispersion, type-I SHG can be described by the following dimensionless partial differential equations

$$\frac{dE_1}{dt} = -(\gamma + i\Delta_1)E_1 + S + E_1^* E_2 + i\left(\nabla_\perp^2 + \frac{\partial^2}{\partial\tau^2}\right)E_1 \quad (1)$$

$$\frac{dE_2}{dt} = -(1 + i\Delta_2)E_2 - E_1^2 + i\left(a\nabla_\perp^2 + \beta\frac{\partial^2}{\partial\tau^2}\right)E_2, \quad (2)$$

The Laplace operator $\nabla_\perp^2$ acting in the transverse plane $\mathbf{r}_\perp = (x, y)$ is $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. The second derivative with respect the fast time τ models the chromatic dispersion. The slow time t is proportional to the round trip time in the cavity. E_1 and E_2 are the normalized slowly varying envelopes of the fundamental and second harmonic intracavity fields at frequency ω and 2ω, respectively. The symbol $*$ denotes the complex conjugate. The input beam amplitude, S, is chosen to be real by fixing appropriately the reference phase and is considered to be constant by assuming continuous-wave operation. The parameter γ is the ratio of the photon lifetimes at the fundamental and second harmonic frequencies. The normalized cavity detunings Δ_1 and Δ_2 represent the distance between the frequency ω and 2ω and the corresponding closest cavity resonance, respectively. The factor $a = 1/2$ multiplying the Laplace operator in Eq. (2) is the ratio of the diffraction coefficients imposed by the phase matching condition. In what follows we fix the ratio of chromatic dispersion coefficients β to $1/2$.

The homogeneous steady states solution of Eqs. (1,2) are

$$(1 + \Delta_2^2)S^2 = I_{1s}\left[(\gamma^2 + \Delta_1^2)(1 + \Delta_2^2) + 2(\gamma - \Delta_1\Delta_2)I_{1s} + I_{1s}^2\right] \quad (3)$$

$$I_{2s} = I_{1s}^2/(1 + \Delta_2^2) \quad (4)$$

with $I_{1s,2s} = |E_{1,2}|^2$. These states are bistable when the following conditions are satisfied $\Delta_1\Delta_2 > \gamma$, $|\Delta_1| > \sqrt{3}\gamma$, and $(|\Delta_1| - \sqrt{3})|\Delta_2| > \gamma + \sqrt{3}|\Delta_1|$.

We perform a linear stability analysis on the steady states. With periodic boundaries in (x, y, τ), the linear deviation from the steady lasing state is proportional to $\exp(\lambda t + i\mathbf{q}.\mathbf{r})$ with $\mathbf{q} = (q_x, q_y, q_\tau)$, where $\mathbf{r} = (x, y, \tau)$ and the wavevector $\mathbf{q}$ verfies the relation $\left(\nabla_\perp^2 + \frac{\partial^2}{\partial\tau^2} + q^2\right) = 0$. This formulation leads to a characteristic polynomial equation that is quartic in λ and whose coefficients are functions of the dynamical parameters of

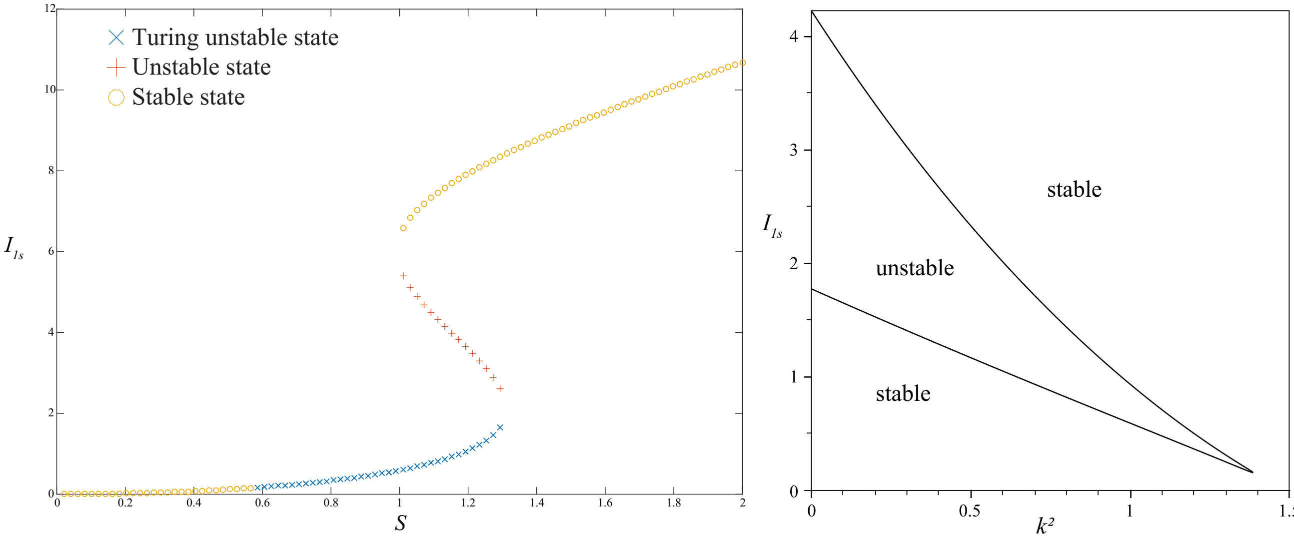

Figure 1. (a) Homogeneous steady states of the fundamental field intensity,I_{1s}, as a function of the input intensity, S^2. Stable states are represented by full line and unstable ones by dashed line. (b)The marginal stabilty curve in the (I_{1s}, q) plane corresponds to unstable modes. When I_{1s} exceeds the critical value I_{1c}, a first mode of wavenumber q_c becomes unstable. Parameters are $\Delta_2 = 2\Delta_1 = -3.$ and $\gamma = 0$.

the system and the wavenumber q^2. For the deviations from the steady states and their complex conjugate, the characteristic equation reads $\sum_{i=1}^{n=4} a_n\lambda^n = 0$. The coefficients a_n depend on the square modulus of the wavevectors q^2. For simplicity, we will focus on the limit of an ideal frequency converter ($\gamma = 0$) and we consider that $\Delta_2 = 2\Delta_1$. These approximations simplify the condition that leads to 3D pattern forming process namely $\lambda = 0$ with a finite wavenumber and hence

$$a_0 = \left[4 - \frac{1 + (2\Delta_1 + q^2/2)^2}{1 + 4\Delta_1^2}\right] I_{1s}^2 - 2(\Delta_1 + q^2)(4\Delta_1 + q^2)I_{1s} + [1 + (2\Delta_1 + q^2/2)^2](\Delta_1 + q^2)^2 = 0 \qquad (5)$$

The plot of this equation provides the marginal stability curve. The critical point associated with a symmetry breaking instability requires an additional condition, namely $\partial I_{1s}/\partial q^2 = 0$. Results of the linear stability analysis for the parameters values $\gamma = 0$ and $\Delta_2 = 2\Delta_1$ are plotted in Figs. (1,2) where examples of marginal stability curves together with the homogeneous steady states are provided. Above the critical instability point there exist a band of unstable modes that triggered the spontaneous formation of three dimensional structures in the Euclidian space (x, y, τ) with an intrinsic wavelength ($\Lambda_c = 2\pi/q_c$) that is determined by the dynamical parameters and not by the system boundaries. Classification of different three dimensional structures are discussed in the next section.

3. THREE-DIMENSIONAL STRUCTURES

Near the three-dimensional symmetry braking instability the solution of Eqs. (1,2) is a linear superposition of l pairs of opposite critical wavevectors $\mathbf{q_j}$ lying on a sphere of radius q_c .

$$(X, U, Y, V) = \mathbf{e_c}\sum_{j=1}^{j=l}[A_j \exp(i\mathbf{q_j}.\mathbf{r})\exp(i\phi_j) + cc] \qquad (6)$$

where X and U are the real and imaginary parts of the field E_1, respectively. Y and V are the real and imaginary parts of the field E_2, respectively. cc denotes the complex conjugate. $\mathbf{e_c}$ is the eigenvector of the linearized operator of Turing mode obtained at the critical bifurcation point. ϕ_j is the phase associated with the mode q_j. The 3D stripes (or lamellae) are characterized by $l = 1$. A lattice of rhombic cells corresponds to

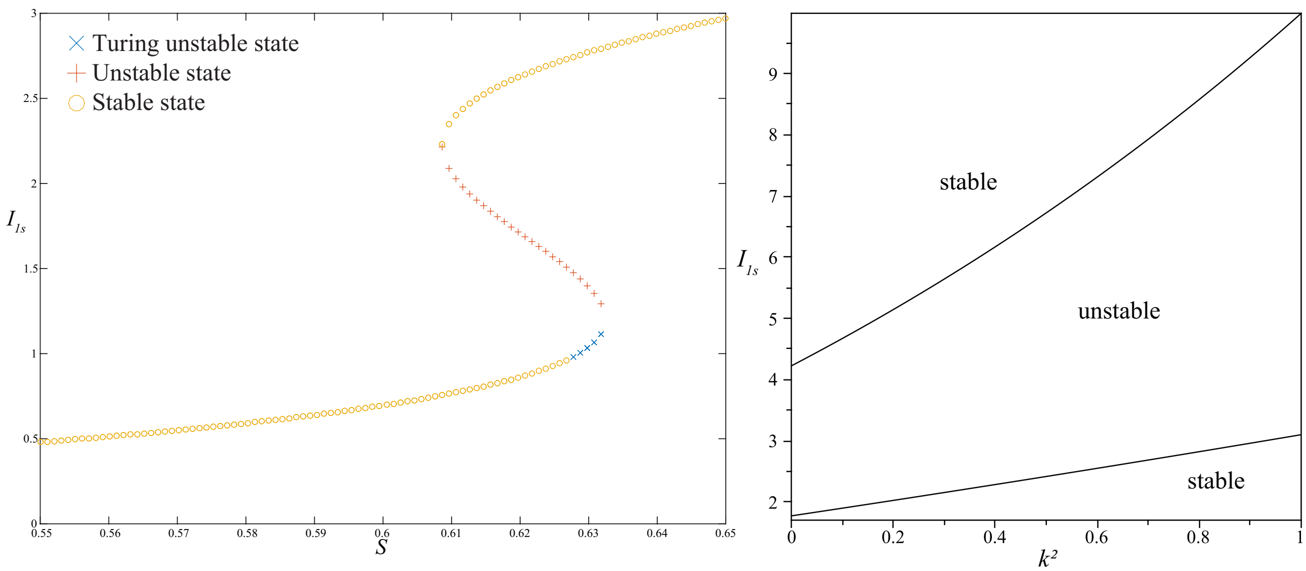

Figure 2. (a) Homogeneous steady states of the fundamental field intensity, I_{1s}, as a function of the input intensity, S^2. Stable states are represented by full line and unstable ones by dashed line. (b)The marginal stabilty curve in the (I_{1s}, q) plane corresponds to unstable modes. When I_{1s} exceeds the critical value I_{1c}, a first mode of wavenumber q_c becomes unstable. Parameters are $\Delta_2 = 2\Delta_1 = 3.$ and $\gamma = 0.$.

$l = 2$. The lattice of hexagonal cells (or hexagonally packed cylinders) is obtained for $l = 3$ with the resonant condition $\mathbf{q_1} + \mathbf{q_2} + \mathbf{q_3} = 0$. The case $l = 4$ corresponds in real space to a face-centred cubic (FCC) lattice. They are intrinsically unstable.[25] The case $l = 5$ corresponds to quasi-periodic crystals and is not treated here. The body-centred cubic (BCC) lattice is obtained for $l = 6$. The coordinate of its six wavevectors in q-space being given by $q_c\sqrt{2}/2(1, \pm 1, 0)$, $q_c\sqrt{2}/2(\pm 1, 0, 1)$, and $q_c\sqrt{2}/2(0, -1, \pm 1)$.

In what follows, we limit our analysis to numerical simulations of the model Eqs. (1,2) with periodic boundary conditions in all three dimensional coordinates. The simplest 3D solutions is the lamelae and hexagonally packed cylinders shown in Figs. 3 and 4. These solutions consist of three dimensional structures travelling at the group velocity of light within the cavity. They coexist in the same parameter range.

4. CONCLUSIONS

We have considered a diffractive and dispersive type one second harmonic generation. We have focused our analysis in regimes where one or more steady states exhibit a three dimensional pattern forming instability. The origin of this instability is the competition between the combined action of both 2D diffraction and chromatic dispersion with a quadratic nonlinearity. In the present work, we have shown the existence of stable 3D localized structures consisting of lamellae, and hexagonally packed cylinders. Some of these structures have been previously reported in the special case of nascent bistability where the dynamics is described by the Swift-Hohenberg equation.[24] In this work, we have studied the full type-I second harmonic generation assuming the mean field approximation.

In the future work we will incorporate a delay feedback loop in our system. This problem has been investigated in 2D diffractive systems.[26–32] We will adress in the near future the problem of the effect of delay feedback on the formation of three-dimensional dissipative structures.

5. ACKNOWLEDGEMENTS

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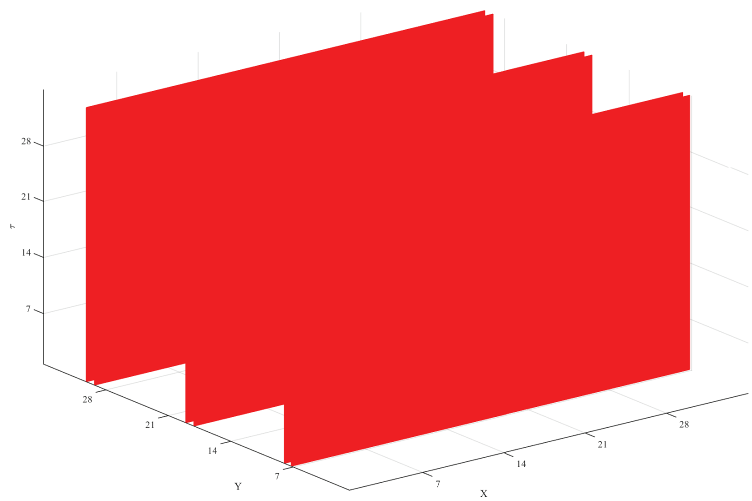

Figure 3. Isosurfaces in the (X, Y, τ) space of the quantity $I_{1s} = 2.75$. Parameters are $\gamma = 0$, $\Delta_1 = 1$, $S = 0.6195$, and $\Delta_2 = 2$. Simulations have been performed using a Runge-Kutta of order 4 method for the temporal integration, and a finite difference method of order 2 for the spatial integration. Simulations have been performed on a $50 \times 50 \times 50$ grid, with a space step of 0.7 and a time step of 0.01.

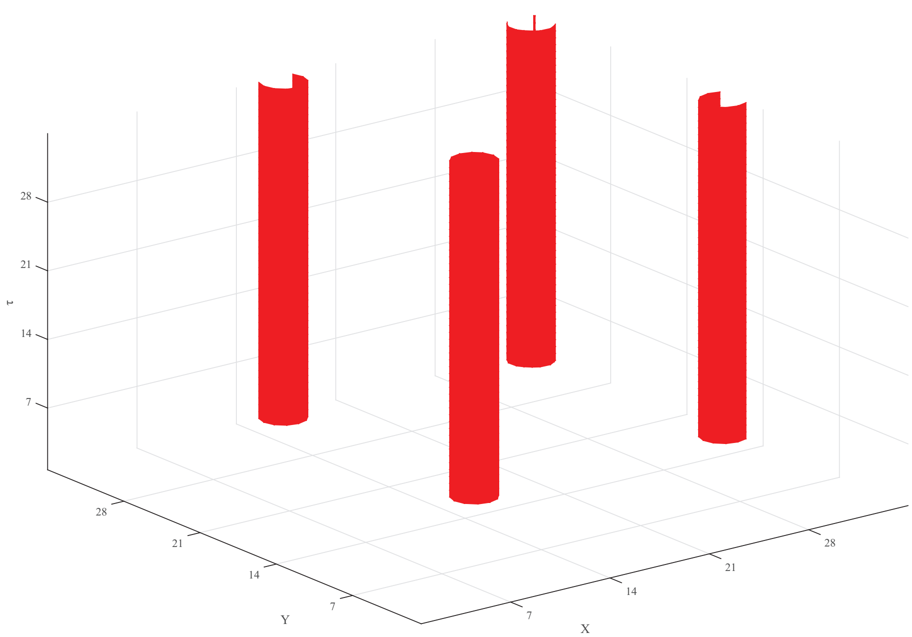

Figure 4. Isosurfaces in the (X, Y, τ) space of the quantity $I_{1s} = 2.55$. Parameters are $\gamma = 0$, $\Delta_1 = 1$, $S = 0.6195$, and $\Delta_2 = 2$. Simulations have been performed using a Runge-Kutta of order 4 method for the temporal integration, and a finite difference method of order 2 for the spatial integration. Simulations have been performed on a $50 \times 50 \times 50$ grid, with a space step of 0.7 and a time step of 0.01.

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